## **HOMEWORK 10**

This problem set is due Wednesday December 2. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding. You may assume that the ground field is the complex numbers.

**Problem 0.1.** Let C be the smooth, complex projective curve associated to the affine plane curve  $y^2 = x^3 + 1$ . Let  $\pi : C \to \mathbb{P}^1$  be the projection to the x-axis. Let  $w = e^{2\pi i/3}$  and let  $p_j = (-w^j, 0)$  for j = 0, 1, 2. Let  $q_j = (0, (-1)^j)$  for j = 0, 1. Let  $r = \pi^{-1}(\infty)$ . Let  $s_j = (2, (-1)^j 3)$  for j = 0, 1. Prove that the following divisors are linearly equivalent:

$$\begin{array}{l} 2p_0 \sim 2p_1 \sim 2p_2 \sim q_0 + q_1 \sim s_0 + s_1 \sim 2r \\ \\ p_0 + p_1 + p_2 \sim 3r \\ \\ q_0 + s_0 \sim q_1 + s_1 \end{array}$$

Determine the complete linear system  $L(p_0 + q_0)$ . Find a point p such that  $p \sim p_0 + q_1 - r$ . Find a point p such that  $p \sim 2s_0 - r$ . What is the genus of C?

**Problem 0.2.** Let C be the plane curve determined by the equation  $x^4 + y^4 - z^4 = 0$ . Let p = (0, 1, 1)Describe the complete linear system L(3p). Let q = (1, 0, 1). Find a pair of points  $r_1, r_2$  such that  $3p \sim q + r_1 + r_2$ . Show that  $4p \sim 4q$ . Describe the complete linear system L(4p). What is the genus of C?