$^{\perp}N$ AS AN ABSTRACT ELEMENTARY CLASS

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We show that the concept of an Abstract Elementary Class (AEC) provides a unifying notion for several properties of classes of modules and discuss the stability class of these AEC. An abstract elementary class consists of a class of models $\boldsymbol{K}$ and a strengthening of the notion of submodel $\prec_{\boldsymbol{K}}$ such that $(\boldsymbol{K}, \prec_{\boldsymbol{K}})$ satisfies the properties described below. Here we deal with various classes $({}^{\perp}N, \prec_N)$; the precise definition of the class of modules ${}^{\perp}N$ is given below. A key innovation is that $A \prec_N B$ means $A \subseteq B$ and $B/A \in {}^{\perp}N$. Here is a sample result.

Theorem 0.1. (1) *Let R be any ring and N an R–module. If $({}^{\perp}N, \prec_N)$ is an AEC then N is a cotorsion module. Conversely, if N is pure–injective, then the class $({}^{\perp}N, \prec_N)$ is an AEC.*

(2) *Let R be a right noetherian ring and N be an R–module of injective dimension ≤ 1. Then the class $({}^{\perp}N, \prec_N)$ is an AEC if and only if ${}^{\perp}N$ is closed under direct limits (of arbitrary homomorphisms).*

(3) *Let R be a Dedekind domain and N an R–module. The class $({}^{\perp}N, \prec_N)$ is an AEC if and only if N is cotorsion.*

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