

Definable sets generated by standard part sets

Let R be a sufficiently saturated o-minimal expansion of a real closed field, set $\mathcal{O} := \{r \in R : |r| < m \text{ for some } m \in \mathbb{N}\}$, let $\text{st} : \mathcal{O} \rightarrow \mathbb{R}$ be the standard part map (it induces for every n a map $\text{st} : \mathcal{O}^n \rightarrow \mathbb{R}^n$), and for $X \subseteq R$ define $\text{st}(X) := \text{st}(X \cap \mathcal{O})$. Let \mathbb{R}_{ind} be the field \mathbb{R} expanded by all sets of the form $\text{st}(X)$, where $X \subseteq R^n$ is definable in R , $n = 1, 2, \dots$. It was observed by Hrushovski, Peterzil and Pillay that the structure \mathbb{R}_{ind} is o-minimal as a consequence of a theorem by Baisalov and Poizat. Without using this theorem we obtain the following stronger version of this fact: the subsets of \mathbb{R}^n that are definable in \mathbb{R}_{ind} are exactly the finite unions of sets of the form $\text{st}(X) \setminus \text{st}(Y)$, where $X, Y \subseteq R^n$ are definable in R . We also discuss some consequences of the (proof of the) above.