

Math 435 Number Theory I
Problem Set 10

Due: Friday November 18

1)a) Compute $\left(\frac{11}{17}\right)$ using Euler's criterion **and** using Gauss' Lemma.

b) Compute $\left(\frac{397}{1231}\right)$.

2) For which primes p is 11 a quadratic residue?

3) Are there any solutions to the equation

$$X^2 + 3X - 7 \equiv 0 \pmod{143}?$$

[WARNING: 143 is not prime]

4) Prove there are infinitely many primes $p \equiv 1 \pmod{3}$. [Hint: Consider $N = (2p_1 \cdot p_k)^2 + 3$ where p_1, \dots, p_k are congruent to 1 mod 3. Suppose p is a prime dividing N . Show $\left(\frac{p}{3}\right) = 1$.]