MTHT 530 Analysis for Teachers II

Midterm I Study Guide

The webpage

`http://www.math.uic.edu/∼marker/mtht530/concepts.html`

contains a summary of all key concepts and results we have discussed in the course.

The webpage

`http://www.math.uic.edu/∼marker/mtht530/wtw.html`

is a summary of what we have covered weak-by-weak.

Sample Exam Questions

The following are the type of questions I will ask on the exam. There are many more quesitons here than I would put on the exam.

1) Define the following concepts

a) f is differentiable at a

b) $(a_n)_{n=1}^{\infty}$ converges to a

c) $\lim_{x \to a} f(x) = l$.

2) a) State the Completeness Axiom

b) State the Bolzano–Weierstrass Theorem

c) State the Mean Value Theorem

3) State and prove the Monotone Convergence Theorem.

4) Decide if the following statements are TRUE or FALSE. If FALSE, give an example showing the statement is FALSE.

a) If $A \subseteq \mathbb{R}$ is bounded and nonempty there is $\alpha \in A$, a least upper bound for A.

b) If $f : (a, b) \to \mathbb{R}$ is continuous, then it is bounded.

c) If $f : [a, b] \to \mathbb{R}$ is continuous and $f(x) \neq 0$ for all $x \in [a, b]$, then $g(x) = \frac{1}{f(x)}$ is bounded on $[a, b]$.

d) If $\lim_{x \to a} f(x) = b$ and $\lim_{x \to b} g(x) = c$, then $\lim_{x \to a} g(f(x)) = c$.

e) If $(a_n)_{n=1}^{\infty}$ converges, then it is bounded.

f) If f is differentiable at a, then f is continuous at a.

g) If $f : [0, 1] \to [0, 2]$ is continuous, there is $x \in [0, 1]$ with $f(x) = 2x$.

5) Suppose $\lim\limits_{x \to a} f(x) = l$ and $(a_n)_{n=1}^{\infty}$ converges to a where no $a_n = a$. Prove that $(f(a_n))_{n=1}^{\infty}$ converges to l.

6) Let $f(x) = x^2$ **Using the definition of derivatives** prove that $f'(a) = 2a$.

7) Suppose $f : [0, 1] \to [0, 1]$ is differentiable and $|f'(x)| < 1$ for all $x \in [0, 1]$. Prove that there is **exactly** one $a \in [0, 1]$ with $f(a) = a$.

8) Let

$$f(x) = \begin{cases} 8x & \text{if } x \in Q \\ 2x^2 + 8 & \text{if } x \notin \mathbb{Q} \end{cases}.$$

Prove that f is continuous at 2, but not at 1.

9) Suppose $f'(x) \geq M$ for all $x \in [a, b]$. Prove that $f(b) \geq f(a) + M(b - a)$.