HOMEWORK 12

This problem set is due Monday November 24. You may work on the problem set in groups; however, the final write-up must be yours and reflect your own understanding. You may assume that the ground field is the complex numbers.

Problem 0.1. *Let C be the smooth, complex projective curve associated to the affine plane curve $y^2 = x^3+1$. Let $\pi : C \to \mathbb{P}^1$ be the projection to the x-axis. Let $w = e^{2\pi i/3}$ and let $p_j = (-w^j, 0)$ for $j = 0, 1, 2$. Let $q_j = (0, (-1)^j)$ for $j = 0, 1$. Let $r = \pi^{-1}(\infty)$. Let $s_j = (2, (-1)^j 3)$ for $j = 0, 1$. Prove that the following divisors are linearly equivalent:*

$$2p_0 \sim 2p_1 \sim 2p_2 \sim q_0 + q_1 \sim s_0 + s_1 \sim 2r$$
$$p_0 + p_1 + p_2 \sim 3r$$
$$q_0 + s_0 \sim q_1 + s_1$$

Determine the complete linear system $L(p_0 + q_0)$. Find a point p such that $p \sim p_0 + q_1 - r$. Find a point p such that $p \sim 2s_0 - r$. What is the genus of C?

Problem 0.2. *Let C be the plane curve determined by the equation $x^4 + y^4 - z^4 = 0$. Let $p = (0, 1, 1)$ Describe the complete linear system $L(3p)$. Let $q = (1, 0, 1)$. Find a pair of points r_1, r_2 such that $3p \sim q + r_1 + r_2$. Show that $4p \sim 4q$. Describe the complete linear system $L(4p)$. What is the genus of C?*

Problem 0.3. *Let C be the smooth, complex projective curve associated to the affine plane curve $y^2 = x^6 - 1$. Let $w = e^{\pi i/3}$. Let $p_j = (w^j, 0)$ for $j = 0, 1, \ldots, 5$. Let $\pi : C \to \mathbb{P}^1$ be the projection to the x-axis. Let $q + r = \pi^{-1}(\infty)$. Prove the following linear equivalences*

$$2p_j \sim q + r$$
$$p_0 + p_1 + \cdots + p_5 \sim 3q + 3r$$

Describe the complete linear system $L(p_0 + p_2 + p_4)$. Find an effective divisor D such that $p_1 + D \sim p_0 + p_2 + p_4$. What is the genus of C?