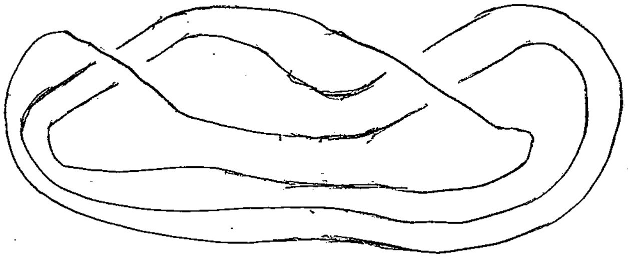
Problem Set #1 – Math 569

1. (a) Show by using Reidemeister moves that $K_{3,2} \cong K_{2,3}$.



$K_{3,2}$

$K_{2,3}$.

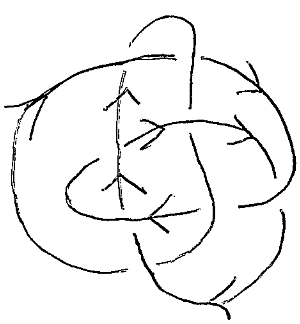
(b) Show by using Reidemeister moves that $E \cong E^*$.

E

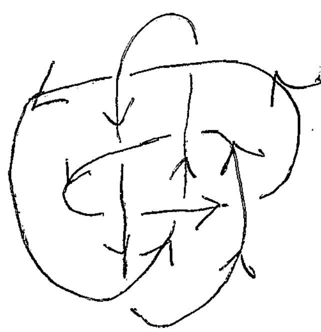
E^*

[K^* = mirror image diagram obtained by switching all the crossings]

(c) Show by using Redemeister moves that $\overrightarrow{8_{17}} \cong \overleftarrow{8_{17}^*}$. (It turns out that $\overrightarrow{8_{17}} \not\cong \overleftarrow{8_{17}}$.)



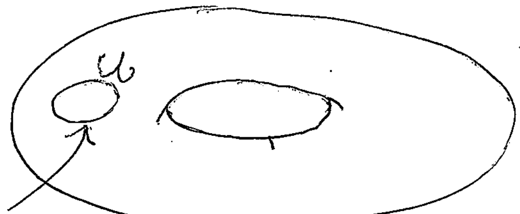
$\overrightarrow{8_{17}}$



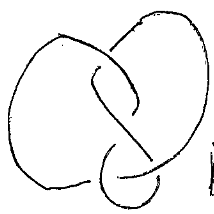
$\overleftarrow{8_{17}}^*$

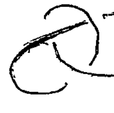
(d) Prove that $(S^3, K_{n,m}) \cong (S^3, K_{m,n})$ for $\gcd(m,n) = 1$ (torus knots).

2. Think of $K_{m,n} \subset S^1 \times S^1$ as a curve embedded in the torus $S^1 \times S^1$. (we take $\gcd(m,n) = 1$). Show that there exists an orientation preserving surface homeomorphism $h : S^1 \times S^1 \circlearrowleft$ such that $h(S^1 \times S^1, K_{m,n}) = (S^1 \times S^1, \mathcal{U})$ where

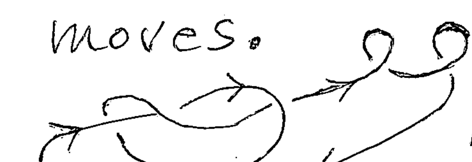
$(S^1 \times S^1, \mathcal{U}) =$ 

trivial circle $\mathcal{U}$ on a torus.

3.  E

Compute a presentation for $\pi_1(E) \equiv \pi_1(S^3 - E)$, and use your result to prove that $E \not\cong T$  $T = K_{2,3}$.

4. Prove that the Wirtinger presentation for a knot diagram K gives a group G that is invariant (up to isomorphism) under the Reidemeister moves.

5.  K

a) Compute a presentation for $\pi_1(M^3(K))$.

b) Show that $H_1(M^3(K)) \cong$ trivial group $\cong \{\phi\}$.

[$M^3(K)$ is the famous Poincaré Manifold]