Math 215 - Assignment Number 5, Spring 2013

Read Chapters 10,11,12. Read the notes on sets by Rudin that we have on our website.

1. Eccles page 115-119. problems 3,5,11,12,13,14,15,16,22.
2. Prove that the number of subsets of the set $\{1, 2, \cdots, n\}$ is 2^n. You may wish to do this by induction on $n = 1, 2, 3, \cdots$.
3. Prove again that the number of subsets of the set $\{1, 2, \cdots, n\}$ is 2^n, this time by using the binomial theorem for $(x + y)^n$ with $x = y = 1$.
4. Eccles page 132. problem 10.3.
5. Eccles page 155. problem 12.2 and problem 12.5.
6. The following is a "proof" that "All horses have the same color." What is wrong with this proof?

Theorem. All horses have the same color.
Proof. We will prove this theorem by induction on n where n is the number of horses. For $n = 1$ we have one horse and obviously this horse has one color. (We assume that each horse has a definite color.). Now suppose for the induction hypothesis that *any k horses have the same color* for some specific natural number k. We wish to show that any $k + 1$ horses have the same color. So let $k+1$ horses be given and choose one of the horses, call it A, and put it aside. We now have k horses and so by the induction hypothesis, they all have the same color. Now take the horse A that we put aside and add it to this group, but take away another horse B. Now the horse A belongs to a group of k horses and so must be the same color as them. But B has this color and so A and B have the same color. We have shown that all the horses except A have the same color and that B (a member of all the horses except A) has the same color as A. Thus all $k + 1$ horses have the same color. We have shown that if every group of k horses have the same color, then every group of $k + 1$ horses have the same color. This completes the induction proof that all horses have the same color. **QED.**

7. The following is a "proof" that $1 = 0$. What is wrong with this proof? Begin with x and y non-zero and $x = y$. Then $x = y$ implies that $x^2 = xy$, and subtracting y^2 from both sides, we have $x^2 - y^2 = xy - y^2$. Now divide both sides by $x - y$ and get $x + y = y$. But since $x = y$ we then have $2y = y$ and since y is non-zero we divide by y and get $2 = 1$. Subtracting 1 from both sides, we have shown that $1 = 0$. **QED.**