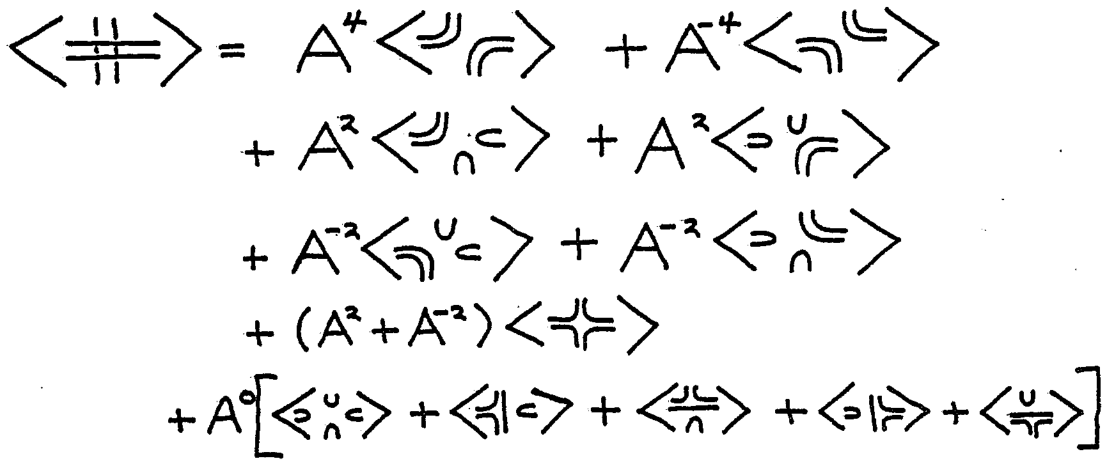
III. Calculating with Double Strands

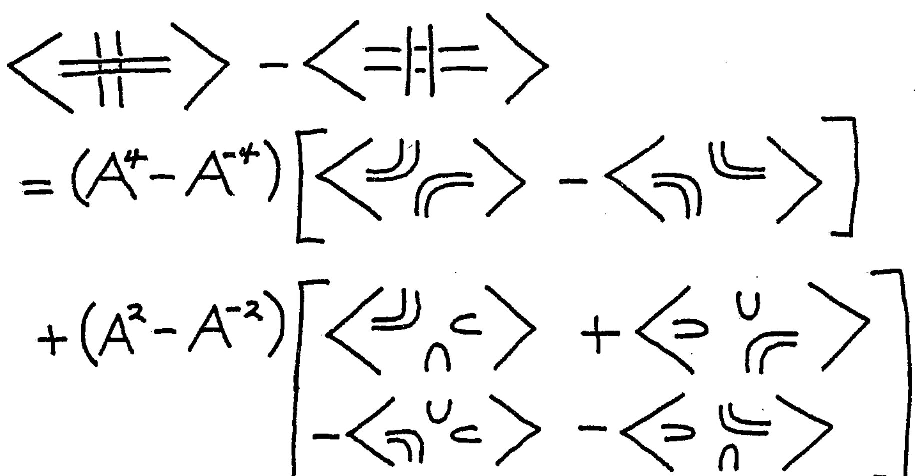
In this section we show how to use the basic expansion formula for the bracket invariant to create a switching calculus for calculating double standed links and doubles of knots. This may be the most practical application of our point of view. It also leads into the possibility of creating many new invariants by generalizing the patterns of these calculi.

By expanding on all four vertices one finds the following formula for the bracket invariant:



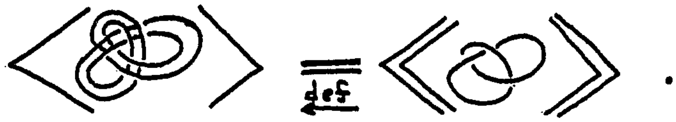
Formula 3.1

This leads immediately to the switching identity

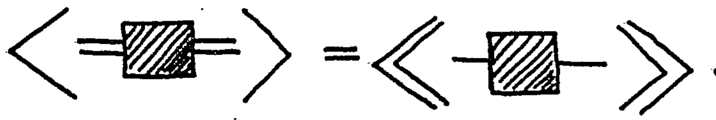


Identity 3.2

With the help of this identity it is an easy matter to calculate <> and hence the Jones polynomial for doubles and double-stranded forms of small knots and links. In doing this

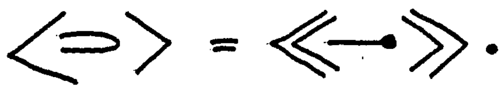


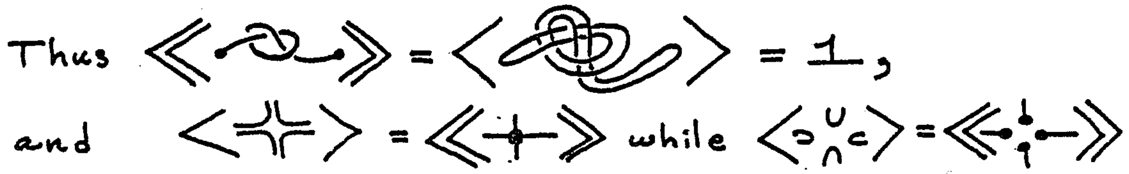
A tangle that needs be kept track of may also be abbreviated. Thus we can write



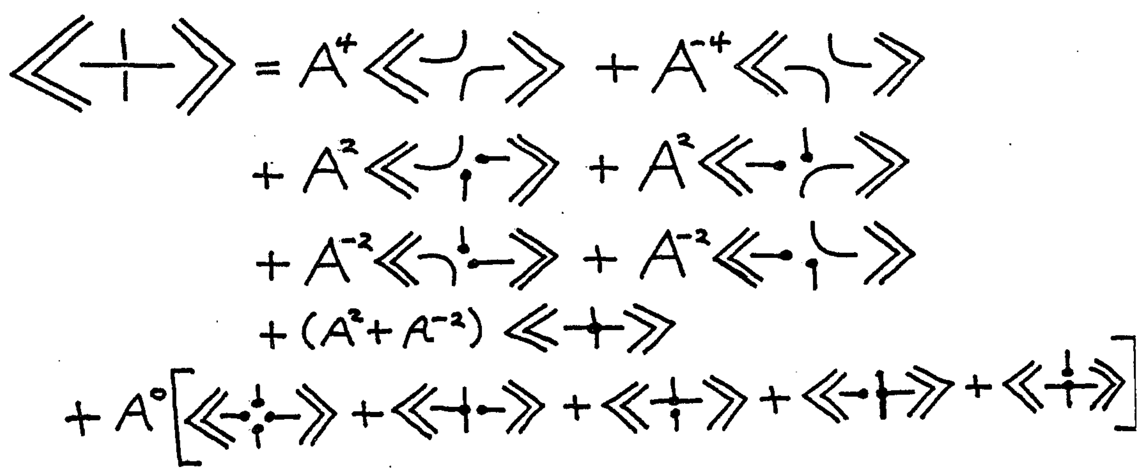
Note that double brackets will always enclose the shorthand.

A further bit of shorthand is

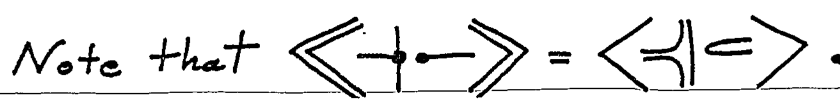


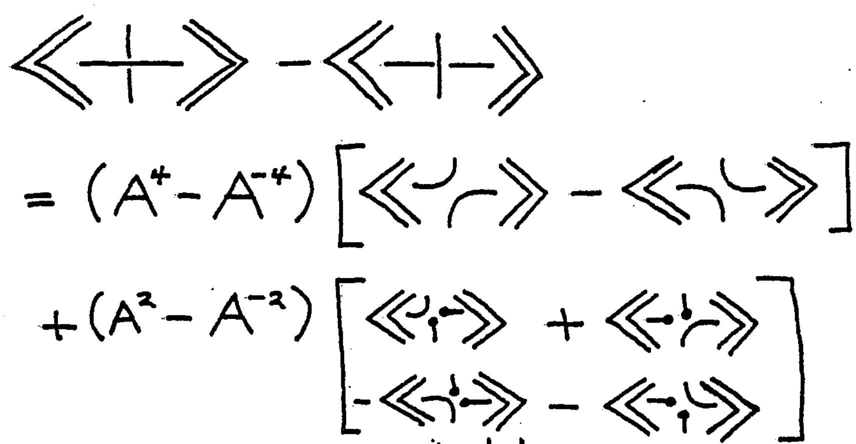


Then our formulas become



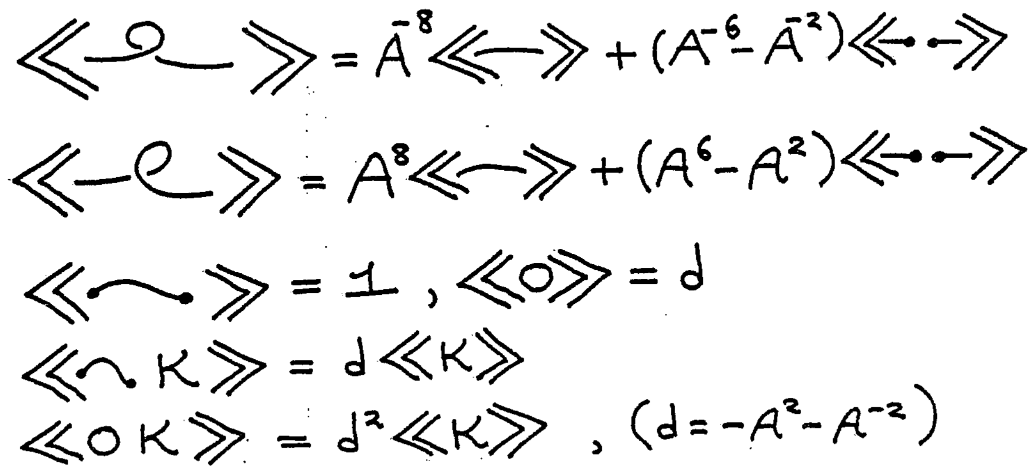
Shorthand Expansion 3.3





Shorthand Identity 3.4

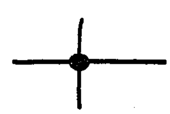
Along with these identities and expansions we find that there are a few further rules that completely facilitate the calculation of bracket for a parallel two-strand knot or link via shorthand.



Further Identities 3.5

The single shorthand strands and diagrams may be changed up to regular isotopy, since regular isotopy of a single strand produces a regular isotopy of the replaced parallels (this is not true of ambient isotopy). The result is that these rules for the shorthand actually produce another invariant of regular isotopy. The new invariant is phrased in a category that contains open strings as well as closed ones, and this means that these formulas implicitly contain rules for dropping certain components.

The shorthand expansion formula **3.3** is a generalized state expansion with the new local state configurations as indicated. It remains to be seen whether other invariants can be constructed from this material. In section 4 we use these ideas but return to the original category of knots and links by including only the one extra local state form



This results in a new invariant. Thus there is indeed a potential here for the construction of invariants through generalized states and the method of undetermined coefficients.

Figure 3.6 illustrates the use of this shorthand.

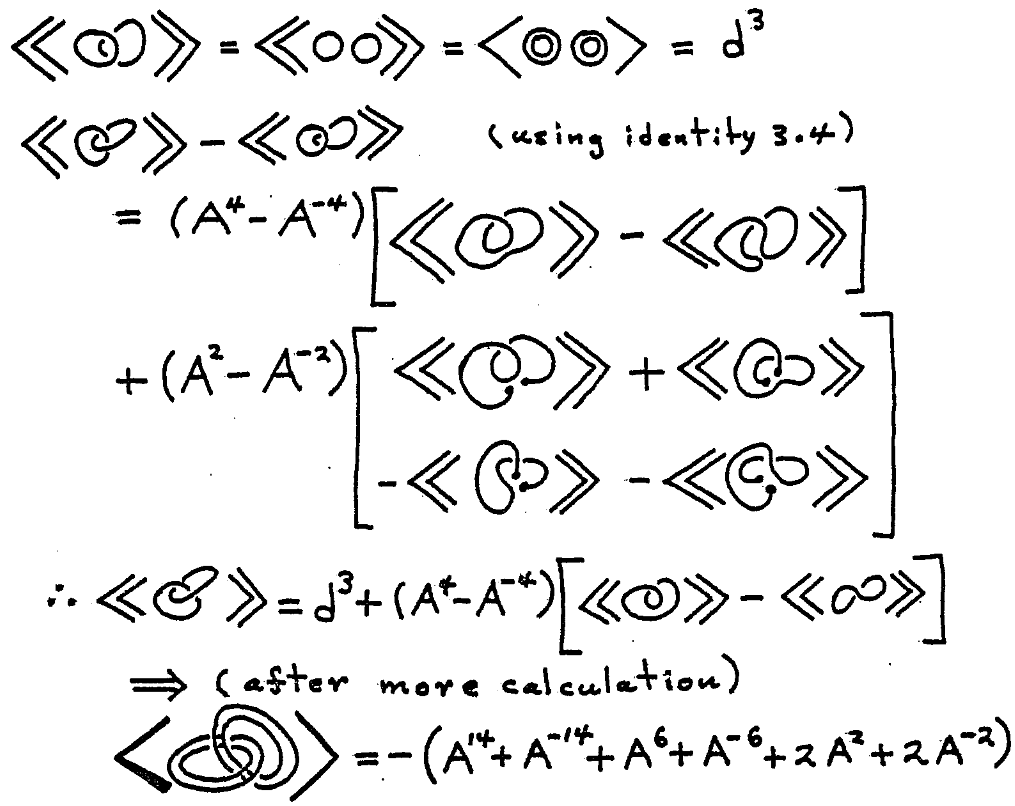


Figure 3.6

We conclude this section with one other useful identity (Figure 3.7). This identity makes it possible to calculate the value of the bracket invariant for ribbon knots and links via the removal of ribbon singularities. Its consequences will be the subject of another paper.

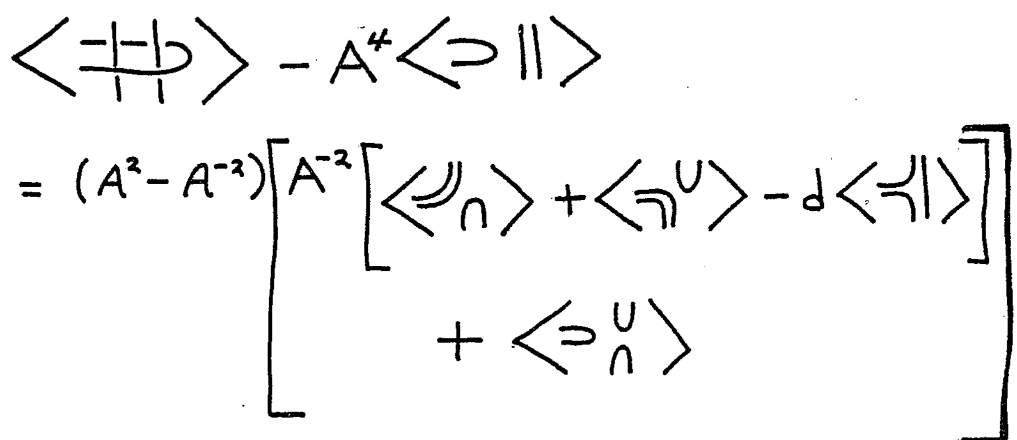
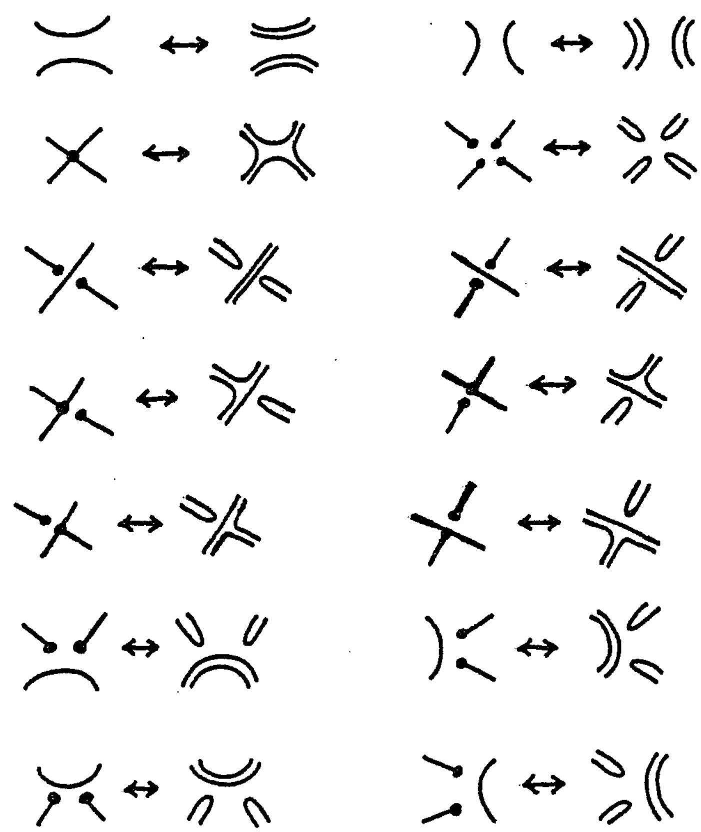


Figure 3.7

In summary , this section has been primarily a catalog of useful formulas and shorthands for calculating the bracket invariant , hence implicitly for calculating the Jones polynomial. These formulas are easily available for the bracket invariant because it has a simple unoriented state expansion. The shorthand can also be viewed as a switching calculus and state expansion for a new regular isotopy invariant in an extended category of knots and strings. This invariant has a state expansion with twelve types of local state configurations. In the next section we shall examine a new invariant that arises through adding only one of these configurations to the states of section 2. In a sequel to this paper we will determine the limitations on invariants that can be built using all these configurations.

<u>Remark</u>. For the full generalization, fourteen local types are required. These, and their double-stranded companions are shown in Figure 3.8.



<u>Local Shorthand</u>

<u>Figure 3.8.</u>