m \ r	17	18	19	20
1	1	1	1	1
2	.0556	.1053	.1500	.1905
3	.0176	.0474	.0857	.1299
4	.0035	.0120	.0260	.0452
5	.0008	.0034	.0085	.0165
6	0	0	0	.0063
7	0	0	0	.0026
8	0	0	0	.0011
9	0	0	0	0

TABLE 3. **Probability of a "dark horse" receiving 17 or more votes under the stated conditions.**

(A simple majority was required for election.) Another fable presented as fact in *The Triple Crown* is discussed in [3].

For a true account of the 1513 conclave, see [4]. The final outcome was that Giovanni de' Medici succeeded in emerging as Pope and reigned until 1521 under the title Leo X.

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An Application of Desargues' Theorem

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Among the highlights of projective geometry is

DESARGUES' THEOREM. *If two triangles are in perspective from a point, the three intersections of the extensions of their corresponding sides lie on one line.*

The purpose of this note is to apply Desargues' theorem to prove the result in Euclidean geometry known as the *Three-circle theorem* [3, p. 115]. The relation of projective geometry to Euclidean geometry makes Desargues' theorem, when properly interpreted, a useful tool in Euclidean proofs. This fact was exploited, for example, by the French geometer Poncelet [4].

In [1], Coxeter gives a proof of the Three-circle theorem using inversive geometry techniques. He also quotes the philosopher Herbert Spencer on his fascination with "a truth which I never contemplate without being struck with its beauty..." [5]. Though Spencer's public interest in mathematics is noteworthy, his understanding lagged far behind his enthusiasm. In [2], MacKay critiques Spencer's statements on mathematics and he describes the philosopher's mathematical knowledge as "both slender and scrappy."

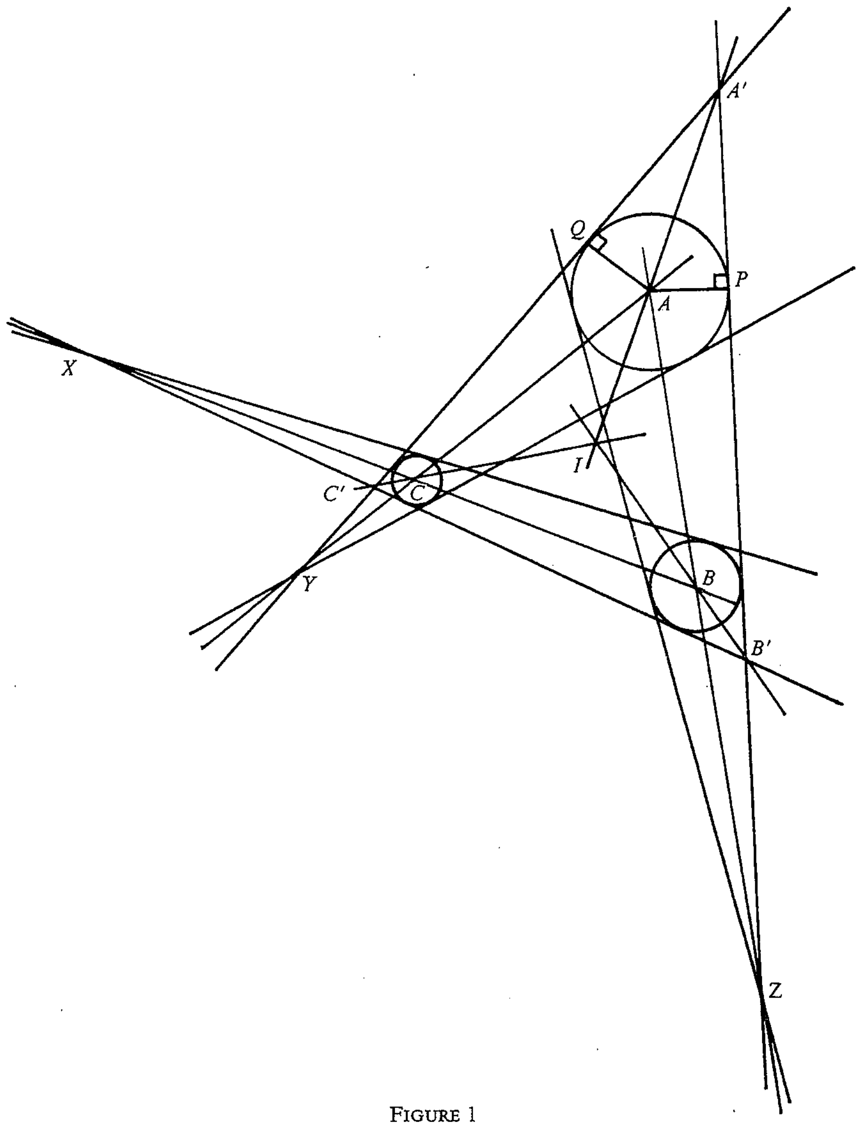

FIGURE 1

THREE-CIRCLE THEOREM. *Given three circles, nonintersecting, mutually external, and with distinct radii, then the three points obtained by taking the intersections of external common tangents of pairs of circles lie on one line.*

Proof. In the proof refer to FIGURE 1. We shall suppose that A, B, and C are the centers of the circles. First observe that the lines joining the centers of the circles also pass through the intersection points of the common tangents.

Consider the triangle $A'B'C'$ obtained from the intersections of tangents external to the triangle ABC. If P and Q denote the points of tangency to the circle centered at A of the lines $A'C'$ and $A'B'$, then triangles $AA'P$ and $AA'Q$ are seen to be congruent right triangles and so AA' bisects angle $B'A'C'$.

Similarly BB' and CC' are angle bisectors of the triangle $A'B'C'$. But the angle bisectors of a triangle are concurrent so AA', BB', and CC' meet at a point I. It follows immediately that triangles ABC and $A'B'C'$ are in perspective from the point I and hence, by Desargues' theorem, X, Y, and Z are collinear.

The reader will want to repeat the proof using the triangle obtained from the intersections of tangents interior to the triangle ABC. Using this interior triangle establishes the theorem in cases when the exterior triangle is degenerate. Furthermore, by introducing internal rather than external common tangents, new collinearities can be found, for example by taking the points of intersection of two pairs of common internal tangents and one pair of common external tangents. By a judicious choice of the intersections of tangent lines, the reasoning in the proof above carries through to prove the collinearity of the corresponding points.

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Mathematician

In midair somewhere
he lays an axiomatic floor.
On it he sets a hypothetical plank
on which he raises a logical ladder
which he proceeds to climb.
    There is risk, suspense and drama:
    any loose rung, any misstep fatal.
At the proper confluence of space and time
he steps off onto a higher platform
with a broader panorama.

The whole thing is fabrication.
But so was Creation.

—KATHARINE O'BRIEN

A version of this poem was read at a meeting of the Poetry Society of America in New York City in February 1981; the program was devoted to poems related to science.

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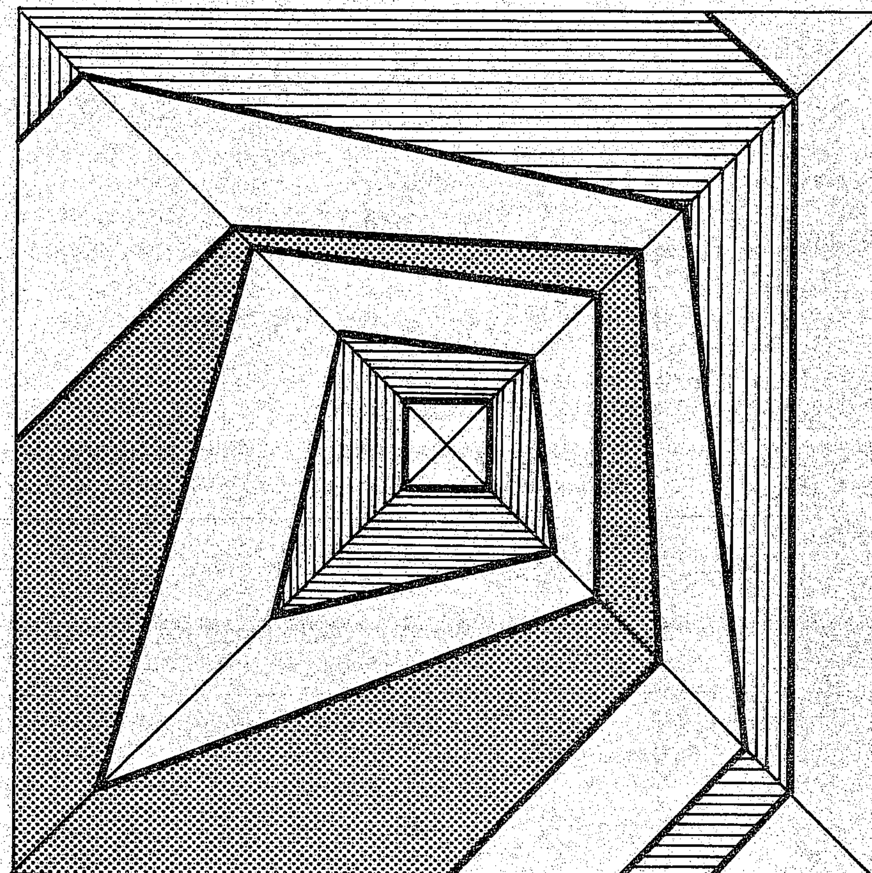

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