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SOME PROBLEMS OF MATHEMATICS AND SCIENCE[1]

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8. **The Wang algebra of networks.** Consider a network of electrical conductors such as shown in the figure. To determine the joint conductance of the network, one could set up Kirchhoff's equations and solve for the current flow through the battery.

K. T. Wang managed an electrical power plant in China, and in his spare time sought simple rules for solving the network equations. Wang's rules were published in the reference indicated below [5]. Wang could not write in English so his paper was actually written by his son, then a college student. Raoul Bott and I recognized that Wang's rules actually define an algebra. We restated the rules as three postulates for an algebra:

$$xy = yx, \qquad x + x = 0, \qquad xx = 0.$$

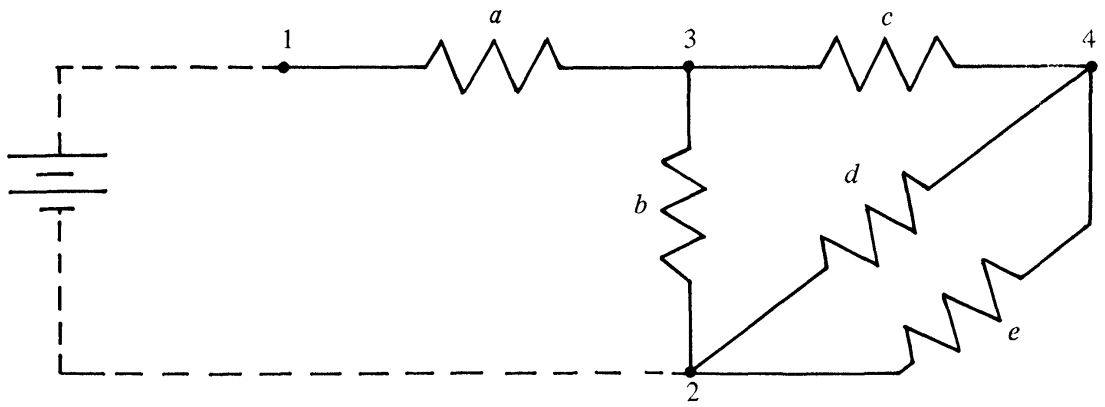

FIGURE 8. A simple network

To apply the Wang algebra to the network shown, let the conductances of the various branches be a, b, c, d and e. Also regard these symbols as independent generators of a Wang algebra. A *star element* of the algebra is defined as the sum of the branches meeting at a node. Thus the star element at node 3 is $a+b+c$.

An algorithm for finding the joint conductance between nodes 1 and 2 follows. First form P, the Wang product of all star elements except those at nodes 1 and 2. Thus

$$P = (a + b + c)(c + d + e).$$

Using the postulate (iii) gives

$$P = ac + ad + ae + bc + bd + be + cd + ce.$$

Next form the Wang product T of all stars except one. Then

$$T = aP = abc + abd + abe + acd + ace. \quad \text{(All trees!)}$$

Then the joint conductance K between nodes 1 and 2 is given as the ratio

$$K = \frac{T}{P} = \frac{abc + abd + abe + acd + ace}{ac + ad + ae + bc + bd + be + cd + ce}.$$

The Wang algebra has interesting and important connections with matroid theory, totally unimodular matrices, and Grassmann algebra. In fact Wang algebra is Grassmann algebra over the mod 2 field. The trees of any graph are given by the above algorithm for T.

1. R. Bott and R. J. Duffin, *On the Wang algebra of networks*, Bull. Amer. Math. Soc. **57** (1951), 136.

2. ———, *On the algebra of networks*, Trans. Amer. Math. Soc. **74** (1953), 99–109. MR **15**, 95.

3. R. J. Duffin, *An analysis of the Wang algebra of networks*, Trans. Amer. Math. Soc. **93** (1959), 114–131. MR **22** #49.

4. ———, *Network models*, Mathematical Aspects of Electrical Network Theory, SIAM-AMS Proc., vol. 3, Amer. Math. Soc., Providence, R.I., 1971, pp. 65–91.

5. K. T. Wang, *On a new method of analysis of electrical networks*, Memoir 2, National Research Institute of Engineering, Academia Sinica, 1934.