FOR NIKLAS LUHMANN: HOW RECURSIVE IS COMMUNICATION?

by Heinz von Foerster *

A year and a half ago Niklas Luhmann sent me a fascinating essay for my 80th birthday (Luhmann 1991). This article culminated in two extraordinary questions. I will not broach these questions in detail now, but I would like to report what kind of an impression these questions made on me. I see in them a similarity to two great problems found in antiquity, two geometric problems. One is the trisection of an angle. That is the problem in which, only with compass and straight edge, one tries to divide an angle into three parts. The other is squaring the circle. The task is, again only with compass and straight edge, to construct a square whose area is the same as a given circle. As you might recall, both of these problems were basically unsolvable, as Karl-Friedrich Gauss proved around 200 years ago. If you take away the constraints of constructing only with compass and straight edge, then these problems can easily be solved.

When I received the invitation to say a few words at Niklas Luhmannn's birthday celebration, I immediately thought, ah, now I'll be able to give the answers to both these problems that he gave me on my birthday. I sat down and worked on these problems, but in the middle of my preparations it suddenly occurred to me: but, Heinz, that is completely wrong! One doesn't do that here in this country. Here they give birthday children questions not answers! So then I thought, good, I'll save my answers for another time. Today I'll come on this occasion to this birthday party with two questions, too. And it won't just concern two questions, since we are here at the Center for Interdisciplinary Research-- but it will concern two research programs about unsolved problems in the social sciences. I thought that today I would present these two problems, because I feel that, if one would concern oneself with these questions, an essential contribution to social theory will be made.

What are these questions? The first problem or rather research program has to do with a furthering or perhaps rather a deepening of the concept of a recursive function. You all know about the unprecedented results of

*<translated by Diane Slaviero and Louis Kauffman>

using recursive functions in chaos theory and other places. But I have the feeling that these results from the chaos theory will only be applied in a metaphorical way to sociological problems. Why? The whole of chaos research uses functions, and functions are only relations between numbers. Consider the function that forms the square of a number. Put in a two, get a four; put in a three, get a nine. It will only operate on numbers. But sociology doesn't work with numbers. Sociology works with functions. And functions of functions are called functors. A functor is a system in which there is a correspondence between one group of functions and another group of functions. And so I propose to develop a research program where one employs recursive functors. I will speak more about it when I get into the details. So that's problem number 1.

Problem number 2 that I would like to discuss is a theory of compositions. It consists of the development of a composition system, and indeed, a composition system for systems. Why consider this problem? I have a system A and a system B, and I would like to integrate both of them into a system C. What are the rules for integration, the composition rules, for allowing a new system C to arise? Is it a matter of addition, of integration? We have nothing but fine words for it, but how is the formalism for such a problem to be expressed? Today one has another name for these composition problems: Consider the Croatians, the Bosnians, the Herzegovinians. One could call it the Vance-Owen problem. This is the problem that we in social theory look at today. How can we solve this problem? The problem can be found in autopoiesis: How can I bring together an autopoietic system A with another autopoietic system B, so that a new system C, also autopoietic, is formed? Unfortunately the poieticists, or the autopoieticists, those who defined autopoiesis, have not given us any rules for the various composition possibilities for such systems. These are, in short, my two problems.

Now you may say, by the will of heaven, we are social scientists and here comes Heinz von Foerster with fundamental mathematical problems, what should we do with them? So I thought that I would sweeten or make the problems milder, in that I would try to present them so clearly as to make them obvious. And when something is obvious, one no longer sees it: The problems disappear. The second idea that I had was to give our birthday child a pair of American gems from California.

The first present is an essay by Warren McCulloch, written about a half a century ago. It is the famous article entitled "A Heterarchy of Values Determined by the Topology of Nervous Nets" (1945). The article is of great importance. I will read a sentence from the last paragraph. The main idea is the circularity of the nervous system: "Circularities in preference". This circularity comes about when A prefers B, B prefers C, and C prefers A. This would be defined as illogical in classical logic. However, McCulloch says that it is not illogical, but that it is how logic is used in reality. So: "Circularities in preference instead of inconsistencies, actually demonstrate consistency of a higher order than had been dreamed of in our philosophy. An organism possessed of this nervous system - six neurons - is sufficiently endowed to be unpredictable from any theory founded on a scale of values". Among existing theories, a system of six neurons was, in principle, previously unheard of. That is present number 1.

Present number 2 that I brought is an article by Louis Kauffman, a mathematician who is fascinated with self-reference and recursion. The article is called "Self-reference and Recursive Forms" (1987). And so that you'll see why I think it's so important, I'll read the last sentence from the article: "Mathematics is the consequence of what there would be if there could be anything at all".

Present number 3 is from my much admired teacher Karl Menger, a member of the Vienna Circle, who I have followed, and still follow, with the greatest pleasure. When I was a young student, I attended Karl Menger's lectures with great enthusiasm. The article by Karl Menger that I bring as present number 3 is "Gulliver in the Land without One, Two, Three" (1959). You might ask why I bring such an article to a sociology group. Karl Menger had already developed the idea of functors, namely functions of functions, which I consider to be quite decisive in the theoretical understanding of social structures. I will also read the last sentence from Karl Menger's article so that you will see what it is about. "Gulliver intended to describe his experiences in the Land without One, Two, Three in letters to Newton, to the successors of Descartes, to Leibniz, and to the Bernoullis. One of these great minds, rushing from one discovery to the next, might have paused for a minute's reflection upon the way their own epochal ideas were expressed. It is a pity that, because of Gulliver's preparations for another voyage, those letters were never written".

So, now I would like to give the three presents to Niklas Luhmann. Naturally this doesn't happen without a bouquet.

Now I come to a topic that was proposed by the Center for Interdisciplinary Research. I always like it when a topic is suggested to me, because then when I come to the meeting, I will completely fulfill the wishes of my hosts. The proposition that has been put forward is the question: "How recursive is communication?" I didn't know how to read that. Should I read it *how* recursive is communication? or How *recursive* is communication? or How recursive *is* communication? Unfortunately I am not an ontologist. That is, I do not know what *is*. So I have framed the question this way: What would it be like if we conceived of communication as recursion? And that is my proposition Number 00:

00. Proposition: "Communication is recursion."

You could understand this as if it were an entry in a dictionary. If you do not know what communication is, just look in the dictionary under c. And there is: "Communication is recursion." Aha, you say, good. What does recursion mean? And then again you go to the dictionary, and look under r: "Recursion is communication." And so it is with all dictionaries. If you work with it, you will soon find that the dictionary is always self-referential: You will be sent from A to B, from B to C, and then from C back again to A. That is the game of the dictionary. You could also choose to see this proposition as simply a tautology: communication is recursion. Yes, but as philosophers maintain, tautologies say nothing. But tautologies do say something about that which they express. At the end of my talk you may perhaps know nothing about recursion or about communication, but you will probably know something about me! My program is then to make the proposition: "communication is recursion".

I would like to divide my program into three parts. In the first part I will cover the essentials, and remind you of the terminology whose central idea is an imaginary "machine" that carries out well-defined operations on numbers, expressions and operations. This part begins with a recapitulation of current concepts. As you will see later, I need these terms in order to take you to the decisive point of my proposition, namely, an understanding of the inherent unsolvability of the so-called "analytical problems". In other branches of science these problems are found under other names: "Decidability problem" in logic, "halting problem" in computer science, etc.

I considered very carefully how to bring you closer to understanding this problem, without recourse to mathematical somersaults. I finally was lead to a compromise position where I will not demonstrate the, in principle, unsolvability of analytical problems, but only a milder version: namely that all the treasures in the world and in our universe and all the available time would be insufficient to solve the analytical problem for even relatively simple "non-trivial" machines. The problem is "transcomputational". Our ignorance is fundamental.

This deep unknowing, this completely fundamental ignorance -- I have never seen developed in its full strength. In view of such fundamental ignorance, how can we work with such problems? In the second part, I will sketch the development of recursive functors. I will make it as easy and playful as possible, so that you will be able to follow the train of thought. In the third part I will speak about composition, compositions of functors and composition of systems.

Part 1: Machines

I begin with the recapitulation a language, introduced by Alan Turing, an English mathematician, in order to shift long deductive, logical arguments to the operation of a machine, a conceptual machine, that will turn all the knobs and gears so that one only needs to watch. Input the problem in one end, and the solution comes out the other end. Once this machine is set up, then it operates a language in which it can very easily spring from one given expression to another. And if you want to know how this machine runs, you can always take it apart. We have a machine programmed by a language of instructions.

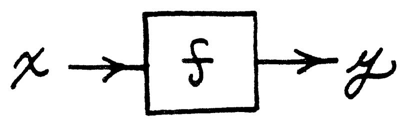
I come to my proposition:

01. Trivial Machines: (i) synthetically determined, (ii) independent of the past, (iii) analytically determined, (iv) predictable.

A trivial machine is characterized by the process of always doing what it was programmed to do. If the machine says it will add 2 to every number, then giving it a 5, a 7 will come out, give it a 10, a 12 comes out. Put the machine on the shelf for a million years and come again and give it a 5 and get a 7, give it a 9 and get an 11. That is the simplemindedness of a trivial machine.

But you do not have to input numbers. You can also give it other forms. For example, in the Middle Ages, logicians gave it logical propositions. The classical logico-deductive proposition is the famous sentence: "All men are mortal". So you come to the "all men are mortal"- trivial machines. Slide in a man in one end, a dead man comes out the other side. Take Socrates "Socrates is a man", push him in one end, boom, out comes Socrates, dead. But you don't need men, and you don't need Socrates, you can also work with the alphabet.

I present an anagram, a machine that computes anagrams. You know that an anagram is something that replaces one letter with another.



$$n_4 = 4! = 24$$

f

x	y
A	B
B	C
C	D
D	A

Figure 1

In order to make this matter as simple as possible, I propose an anagram that has only four letters (A, B, C, D) as in the table in Figure 1: A is replaced by B, B by C, C by D, and finally D by A. When I was a little kid and I wrote letters to my friends, we agreed upon an anagram, naturally so that the parents could not read what we wrote. Of course, these anagrams are very easy to solve. How many anagrams can one build with four letters? As you know, it is simply the number of permutations of the four letters A, B, C and D. It is 4 times 3 times 2, that is 4!, which gives 24 anagrams (Figure 2).

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
A	A	A	A	A	A	B	B	B	B	B	B	C	C	C	C	C	C	D	D	D	D	D	D
B	B	C	C	D	D	A	A	C	C	D	D	A	A	B	B	D	D	A	A	B	B	C	C
D	D	B	D	B	C	C	D	A	D	A	C	B	D	A	D	A	B	B	C	A	C	A	B
D	C	D	B	C	B	D	C	D	A	C	A	D	B	D	A	B	A	C	B	C	A	B	A

The 24 anagrams that are made from exactly 4 letters.

Figure 2

Here are 24 anagrams at my disposal, and if you want to run an experiment to find out which anagram it is, then you only need four trials. You give it an A and get a B; put in a B, get a C; put in a C, get a D; and finally from a D, you get an A. So you have solved the problem. Trivial machines are like that, as formulated in Proposition 01, synthetically determined: we have just built one; time independent: we could put it on the shelf for years; analytically determined: we have just done that; and therefore predictable.

One understands the great love affair western culture has for trivial machines. I could name example after example of trivial machines. When you buy a car, you naturally ask for a trivialization document from the salesman. It says that the car will stay a trivial machine for the next 100 or 1000 miles or the next five years. And if the car suddenly proves itself unreliable, then one gets oneself a trivializer, who will put the car back in order. It goes so far with our love for trivial machines that our children, who are usually very unpredictable and completely surprising fellow creatures, are sent to trivialization institutions, so that if one asks "How much is 2 times 3?" The answer is not "green" or "I am that old" but "6" is the confident reply. And so they will be reliable members of our society.

02. Non-trivial machines: (1) synthetically determined; (ii) ~~in~~dependent ~~of~~ on the past; (iii) analytically indeterminate; (iv) unpredictable.

Now I will speak about non-trivial machines. Non-trivial machines have "internal" states.

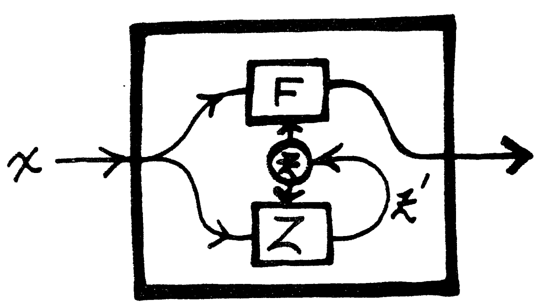


Figure 3

Every operation changes these internal states, so that when the next operation takes place, not only will the previous operation not be repeated, but another operation can be performed in its place. One may ask, how many non-trivial machines can one construct if one has our example of 24 different types of internal states? The number of such possible machines is $N_{24}=6.3 \times 10^{57}$. That is a number followed by 57 zeroes. And you can readily see the difficulties that arise when you want the machine to do analytical investigations. If you ask this machine a question every microsecond, then even if you had all the time in the world it still would not be enough to see through this machine. My next proposition runs thus:

03. Quantities: Let n be the number of inputs as well as the number of outputs. Then the number of possible trivial machines is N_T: $N_T(n) = n^n$ and the number of possible non-trivial machines N_{NT}: $N_{NT}(n) = n^{nz}$, where z is the number of internal states of the NT-machine, and z cannot be greater than the number of possible trivial machines, so: $z_{max} = n^n$, where

$$N_{NT}(n) = n\hat{\ }\{n(n\hat{\ }\{n\})\}.$$

(a^b is here denoted a^{b} to allow iterated exponentiation.)

For a trivial anagram (z=1) of four letters (n=4), this becomes:

$$N_T(4)=4^4=2^{2x4}=2^8=256.$$

For a non-trivial anagram (one which computes various anagrams from previously written rules): NNT(4) = 4^{4(4^{4})}= $2^{2x2x2x256}$ = 2^{2048} about 10^{620}.

W. Ross Ashby, who worked with me in the Biological Computer Laboratory, built a machine with 4 outputs, 4 inputs, and 4 internal states, and gave this machine to the graduate students who wanted to work with him. He told them to figure out how the machine worked and that he would come back the next day. Well, I was a night owl. I would come into the lab around noon and then not go home until 1, 2, or 3 am. These poor students were sitting and working and making tables. I said to them: "Forget it! You can't find it out!" They said: "No, no, I almost have it!" Around 6 am on the next day they were still there working, green and pale. When Ross Ashby came in, he said to them: "Forget it! I'll show you how many possibilities there are: 10^{126}." Then they were quiet.

Imagine that we have only 4 conditions, with input and output symbols and with internal states of 24 possibilities. The complexity of this system is so gigantic, that it is absolutely impossible to ever figure out how this machine works. And then the representatives of "artificial intelligence" dare to say that they are going to find out how the brain functions -- the brain that has 10^{10} neurons at its disposal! They say, "I have worked on a machine that works like the brain." "Oh, congratulations: how does the brain work?" No one knows. One cannot draw a comparison. One can only say, the *machine* works this way and that, but one cannot say how the *brain* works, because no one knows that. But perhaps one does not need to know how the brain works. Perhaps it is like the American proverb: "We are barking up the wrong tree."

For example, how is it possible that this colloquium, in Luhmann's honor, is brought about? Even though we have no idea how the brain works, we all meet here at exactly nine o'clock. And what do we see? Everyone is there, everyone listens, some make sounds with their mouth, others take notes, and so on. So, how is this possible? What is happening here?

From here I would like to develop the next step. As I hope to show you, this only happens because these systems are operating recursively. An astonishing matter!

Part 2: Recursion

In order to make the following thoughts as clear as possible, I will increase the complexity step by step, so that you will be able to follow and know what it concerns.

04. Dimension 1 (operationally open)

I am starting with systems of dimension 1. Why dimension 1? Because here signals are linear and only flow in one direction. One could demonstrate this situation in its utter simplicity through a directed line.

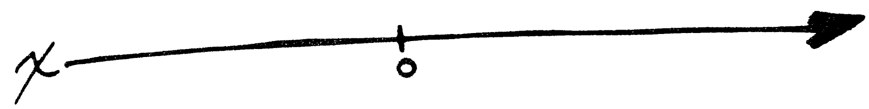


Figure 4

In all operations, which pass through a single point 0, x is transformed into y.

Since I have the intention to speak about compositions of at least two systems, I now present the two systems D and S.

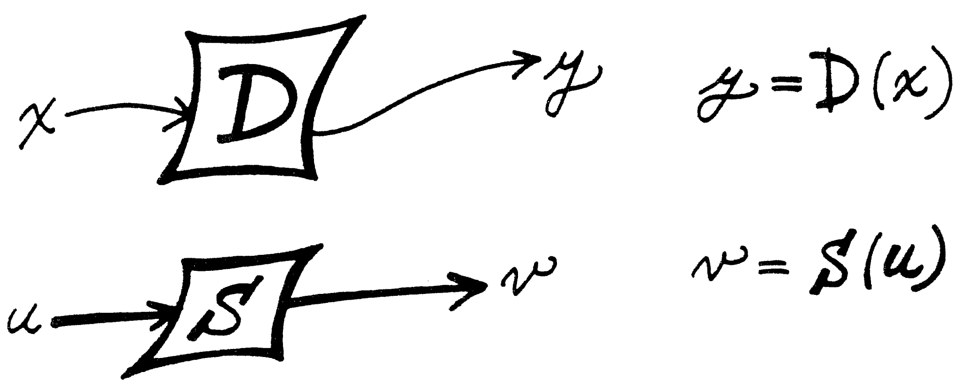


Figure 5

D operates on the variable x and produces y, which is expressed by the function y = D(x). The same applies for machine S, mutatis mutandis.

The letters have a historical background. In the development of recursive machines or non-trivial machines one distinguishes between two functions:

the state function, or S-function, and the driving function, or D-function. For this reason they are called D and S, but you don't need to worry about the D and S. You need only be able to make a distinction between the two machines, one of which operates on x and produces y, while the other operates on u and produces v.

Parametrization

Now from the outside we add to these machines a new control variable so that we can alter the operations of these machines.

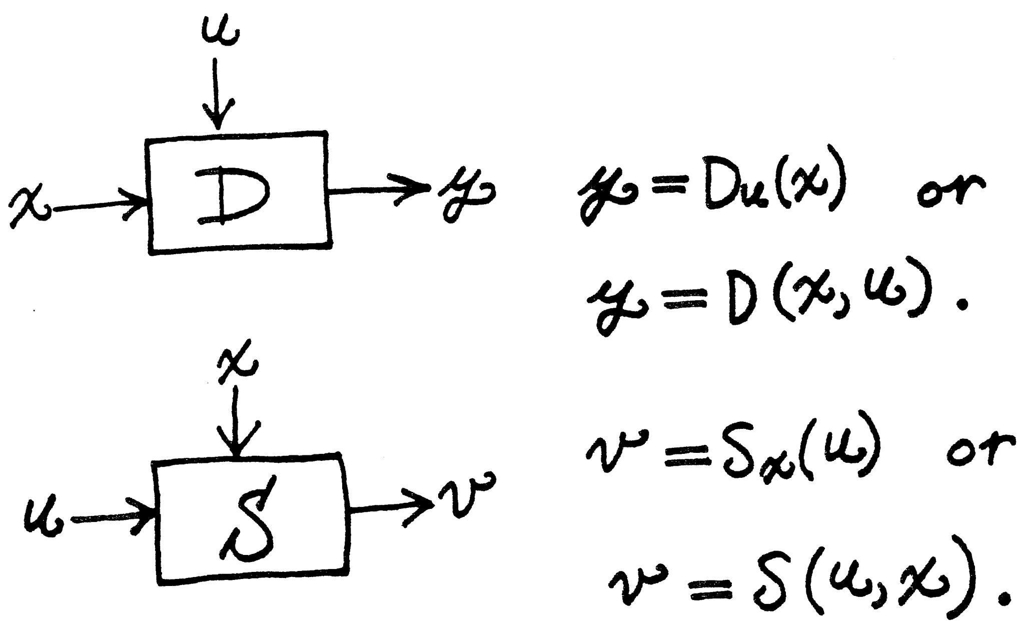


Figure 6

The control functions u and x, which drive the machines from above, should change the operations of the machines. Seen another way, the parameters allow us to control the non-triviality of the machines. If the menu of the 24 anagrams from before were at the machines' disposal, then one could switch, as with TV, from channel to channel; thus here from anagram to anagram. In algebraic terms, this can be expressed in two ways. The parameter can be indicated once through a modified subscript of the function: $y = D_u(x)$, $v = S_x(u)$, or it can be declared as a full-fledged variable: $y = D(x,u)$, $v = S(u,x)$.

04.1. Dimension 2 (operationally closed: the fundamental equations of non-linear dynamics)

Now a decisive step comes, because I shall transform the one-dimensional system into the second dimension through operational closure, by the newly generated output becoming the next input (Figure 7).

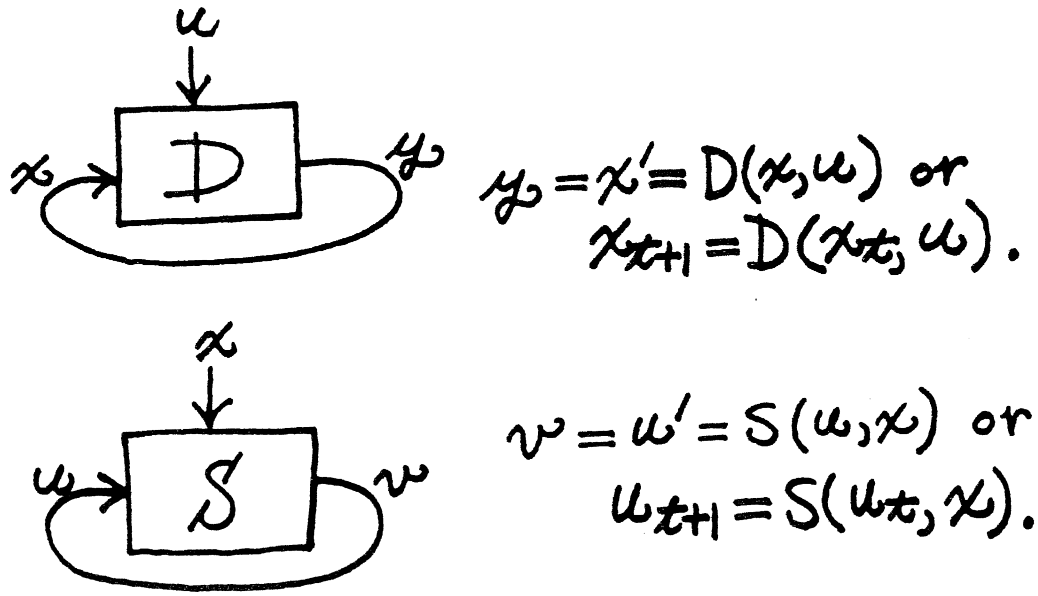


Figure 7

Let y, the output of the D machine, become the input to the S machine. Do the same with the S machine. This step makes a transformation from operational linearity into operational circularity. The situation lends itself to be represented in a plane, that is, on a 2-dimensional manifold.

It can also be expressed as an algebraic formalism in two ways: in one way, the present result of an operation, the output y, is made the next input, x' = y:

$$x' = D(x,u) \quad \text{and} \quad u' = S(u,x).$$

The recursion of these expressions is recognizable. The variables x and u appear as functions of themselves. A "physicalization" of this affair would include the time factor in the expression. The parameters of time are introduced through single steps: now t, and then t+1 a single step later:

$$x_{t+1} = D(x_t,u) \text{ and } u_{t+1} = S(u_t,x).$$

Those of you who work with chaos theory and with recursion will immediately recognize that these are the fundamental equations of recursive function theory. This is the conceptual mechanism underlying chaos research. The equations are always the same. And surprisingly, some unforeseen operational characteristics arise. Historically one has already noted a convergence to certain stable values. For example: recursively take the square root of some favorite number (most calculators have a square root key). Then you will soon stabilize at a value of 1.0000... . No wonder, because the square root of one is one. Mathematicians at the turn of the century, who discovered such stabilities, called these values "eigenvalues" of the corresponding functions. The eigenvalues 1 and also 0, since $\sqrt{0} = 0$, belong to root operations. The essential difference between both of these eigenvalues is in terms of the deviation from the initial value. If you deviate from 1, the recursive nature of the system brings it back to eigenvalue 1. Even the slightest deviation from 0 brings the system back to the stable eigenvalue of 1.

Around twenty years ago, new interest in recursive functions exploded. One discovered that many functions not only had stable *values* but developed stable *dynamics*. These stabilities are known as attractors, evidently a teleological afterthought. One can let a specified system go through the most varied eigenbehaviors by simple changes of the parameters. This pushes one very quickly into the highly interesting behavior that is set in motion by these parametric values: the system passes through a sequence of values that never repeats itself, not even the initial value is repeated: the system becomes chaotic.

Let me make a few remarks about stable eigenbehaviors.

Please observe the next process where, by recursion, only definite discrete values are chosen from an infinite continuum of possibilities. Recall the square root operation where, from the infinity of real numbers, only one, namely the "1", was brought forth. Can that be taken as a metaphor for events of nature, sometimes known as evolution, that from the limitless possibilities for discrete entities, a fly, an elephant, even a Luhmann will be selected? I claim "yes", and I hope to add more bricks to this foundation.

Please also observe that a given set of eigenbehaviors may be closed under a collection of operators, but one cannot necessarily deduce the operators from the eigenbehaviors. For example, the "1" is the eigenvalue of infinitely many different operations. Therefore the recursive eigenvalue "1" of the square root as generator is not the whole story because iteration of the square root function on the value "1" only produces the value "1". In order to see the process, one needs to iterate the operator on a variety of initial conditions. The pure iteration of the operator on itself,

$$\surd(\surd(\surd(\surd(\surd(\ldots))))),$$

symbolizes the process itself without any numerical value. This too, is an eigenbehavior. It is a *pattern* , invariant under an operation on patterns.

Can that explain metaphorically the recursion of natural events, sometimes called nature's laws, of which a limitless number of versions could be given, a Milky Way system, a planetary system, yes even a Luhmann? I assert "yes", and I base this on Wittgenstein's Tractatus P. 5.1361: " Belief in the causal nexus is *the* superstition."

The form of these eigenvalues is the only thing that we can lean on. That is where the impenetrable machine behaves so as to become predictable as soon as it produces the pattern of an eigenform. Then I can naturally say to you, for example, if this eigenform is a periodic sequence, what the next value in the sequence is. By this closed recursive operation and only by this closed recursive operation can one find the stable form: it is not found through an examination of the input and output. It is fascinating that one can observe the stable points, but in principle it is uninvestigatible how these stabilities are produced. One cannot by analysis find out how this system operates, although we see that it operates in a way that makes it possible to make predictions.

Part 3: Compositions

I have spoken of systems as wholes, about their relations, synthesis, analysis and taxonomy. But here I am in the society of learned social scientists. That is the science of "socius", of companions and colleagues, and of the "secundus", of the followers, of the supporters. I have to work with two systems, with their relations, their syntheses and analyses. In fact a society usually contains more than two members, but if the procedure of integration of the "composition " of the two systems is established, then one can apply the established composition rules stepwise and recursively to as many new arrivals as one wants.

How does such a condition come about?

Here, I believe, is the most essential step of the development, because through composition of two systems of dimension 2, the recursive functors arise from irreducibles in the third dimension.

But what is this third dimension?

04.3 Dimension 3 (Calculus of recursive functors)

View Figure 8. I return to the two machines of Figure 7, recursions D and S. In the first step I rotate recursion S by 90°, so that variable and parameter in D and S are oriented parallel to one another; in the second step I push both of them together so that out of the separated systems D and S comes a new machine, a DS-composition.

This DS-composition can be described algebraically by the two equations *taken together*

$$x'=D(x,u)$$
$$u'=S(u,x),$$

where it is understood that the marked values x' and u' are fed back to both D and S. In this way D and S lose their individuality and become contributors to the composite system.

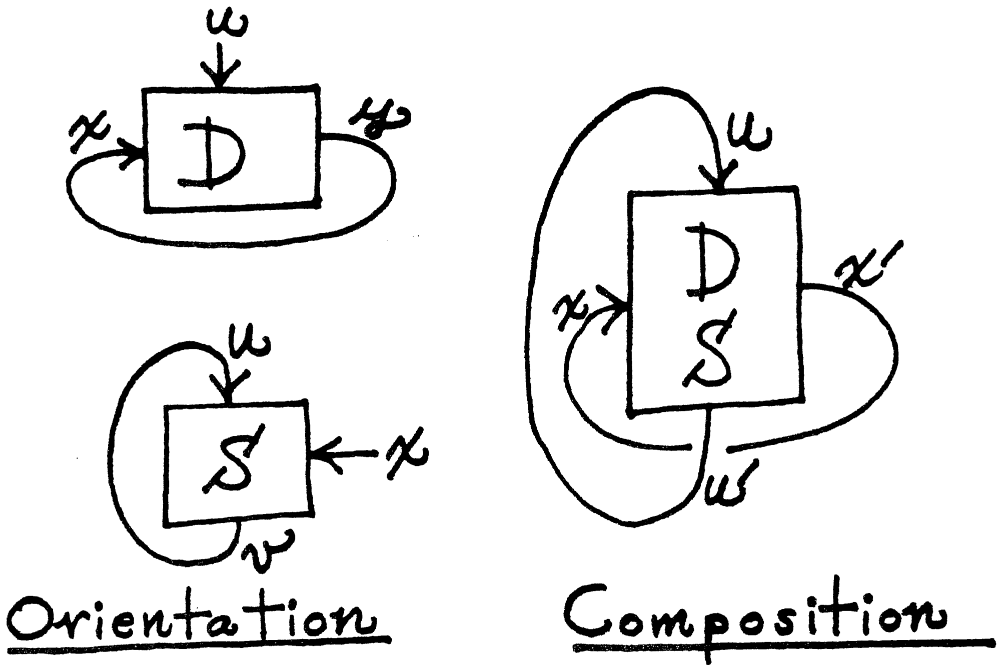


Figure 8

This new machine is drawn with double closures. Now both systems control each other side by side. The operational parameters of each system become functions of the other system: two linked recursive functors.

THE COMPOSITION OF THE SECOND ORDER

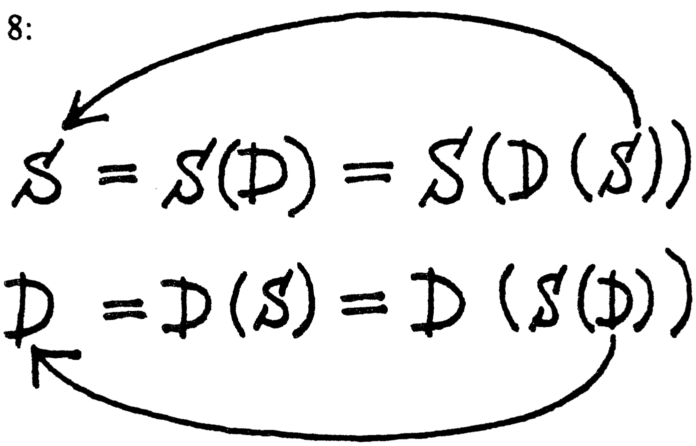
05. Functors: functions of functions (functions of the second order).

From your school years you might recall differential and integral calculus. One writes dy/dx and speaks of "the differential quotient of y by x", where y is a function of x: $y = f(x)$. That means that the differential quotient or differential operator *Diff*, as I would like to call it, is a functor, since it operates on one function, say $y = x^2$, and produces a function: $\mathrm{Diff}[x^2] = 2x$, or in a more elegant quantitative form: $\mathrm{Diff}[()^2]=2$. Does Diff have eigenfunctions? Yes, indeed. Consider the exponential function $y=e^x$, and, following Menger, Diff[exp]=exp. Because of the surprising relationship of the exponential function to the trigonometric functions sine and cosine

$$(\exp(i\Theta)=\cos(\Theta)+i\sin(\Theta) \text{ where } i^2=-1),$$

we have $\mathrm{Diff}^4[\sin]=\sin$, $\mathrm{Diff}^4[\cos]=\cos$. Whence sine and cosine are eigenfunctions of the four times iterated differential operators.

One does not need to constrain oneself to mathematical expressions. Menger (1962) developed these ideas for logical functions into a generalization that is meaningful in wider contexts. For example the algebraic expression of the composition of two systems D and S is rendered clear in Figure 8:



The recursion of functors D and S.

06. Compositions (the characteristics of the composition are not those of the components)

Historically, attention was paid to the qualitative changes which arose in the transition from aggregate to system. By an unlucky description, this transition is identified with the slogan: "The whole is greater than the sum of its parts." New Agers, advocates of holism and environmentalists take to this phrase. As one of my colleagues once remarked: "Can't these fellows add?"

If a density function M is introduced, then the sentiment can be made more precise: " The measure of the sum of the parts is the sum of the measure of the parts": $M\Sigma(T_i)=\Sigma M(T_i)$, (i=1, 2, 3, 4, ... n). If the density function is superadditive, then even the holistic motto, properly formulated, is a legitimate usage. Take two parts a and b, and square the density function, then, in fact, $(a+b)^2$ is greater than $(a)^2 + (b)^2$ because $a^2 + b^2 + 2ab$ is greater than $a^2 + b^2$. To be more exact, the difference is at the place of alternating order, ab + ba, of the system, where, because of symmetry (ab=ba), ab + ba = 2ab.

A first step in the generalization of the density function allows one to write down the rules of the game for a composition algebra, where one looks at the distributive law for the operators as only a special case. Let there be a particular composition K (addition, multiplication, logical implication, etc.), then the failure of the distributive law is expressed by the equation:

$$Op[K(f,g)] \neq K[Op(f), Op(g)].$$

In other words, the result of an operation, Op, on a system built through the K composition is not equivalent to the K composition of the results of Op.

This inequality plays an important role among the autopoieticists, who like to always insist that the properties of the autopoietic system are not able to be expressed through its components.

And now I want to briefly mention two important cases (one is a restriction and the other an extension). Both allow the interchangeability of operator and composition.

(i) **homogeneous composition:** K is said to be a homogeneous composition rule with respect to the operation Op if

$$Op[K(f,g)]=K[Op(f), Op(g)];$$

(ii) **superposition:** A composition rule C is said to be a superposition of Op and K if

$$Op[K(f,g)]=C[Op(f), Op(g)].$$

Note that if Op has an inverse operation, denoted Op^{-1}, then C is indeed obtained from Op and K by the formula

$$C[f,g] = Op[K(Op^{-1}(f),Op^{-1}(g))].$$

The creators of information theory made these formulations, following Boltzmann's example. For the entropy H (here Op) they chose the logarithmic function. The entropy, H(X), is a measure of the uncertainty for a system X. The measure for the uncertainty for two independent systems X and Y, H(X & Y), should be the sum of the measures of uncertainty of the individual systems. This yields the equation H(X & Y) = H(X) + H(Y). For specific numerical measures of the individual systems this leads to the numerical equation $H(xy) = H(x) + H(y)$. This can be represented by the logarithm function since $\log(ab)=\log(a)+\log(b)$.

If you look more closely at the "composition" in Figure 8, you will see, that in principle it is impossible for both the x and the u loops to be in the plane of the paper without cutting each other. One must either raise x or u out of the plane of the paper into the "third" dimension in order to retain the independent paths of both recursions.

This three-dimensionality can be further clarified by drawing the circuit for the combined system on the surface of a torus. Then the u-loop winds around one of the toral directions while the x-loop winds around the other toral direction. The two loops need only meet at the DS-box on the torus surface. This ring or torus is then the topological representation of a doubly closed system.

If you would like pictures, you will find this illustrated already in a paper by Warren McCulloch: "A Heterarchy of Values Determined by the Topology of Nervous Nets":

07. Warren S. McCulloch: "A Heterarchy of Values Determined by the Topology of Nervous Nets". (1945) (Figure 9)

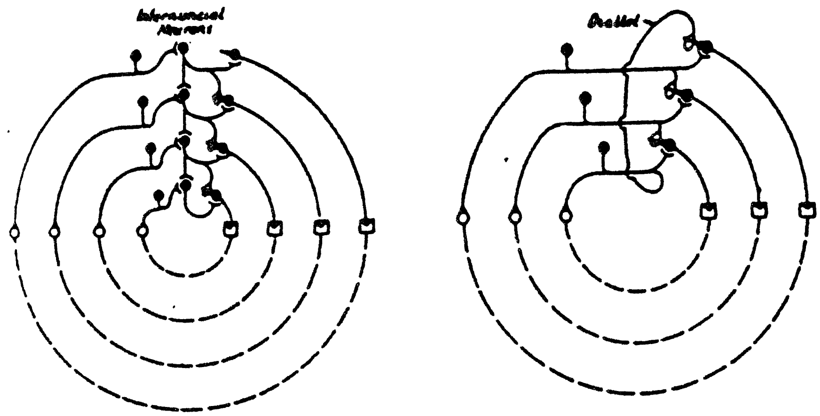


FIGURE 3 FIGURE 4

Figure 9

His argument is the following: In his Figure 3 (our Figure 9) he shows the recursion of neuronal activity, where the internal components are drawn out and the external ones are indicated by the dashed circle segments. This illustrates McCulloch's thesis about environmentally closed nerve pathways. In this network diagram the organization is hierarchical, because the outer sensory motor loops (dromes) could inhibit the inner ones. Therefore this network cannot compute the "circularities in preference", the "value-anomaly" which I already mentioned. In McCulloch's Figure 4 he introduces the diallels which are neurons that can inhibit the outer circle from the inner one. The result is a three-dimensional double closure in which the bottom of the planar hierarchy can influence the top of that hierarchy. In three dimensions the hierarchy is toppled and heterarchy emerges.

Another point about the usefulness of the toroid in the presentation of the double closure process is found in proposition 08:

08. Double Closure of the sensory motor and innervated neuronal circle (N = neural bundle, syn = synapses, NP = neuropituitary, MS = motor surface, SS = sensory surface). (Figure 10)

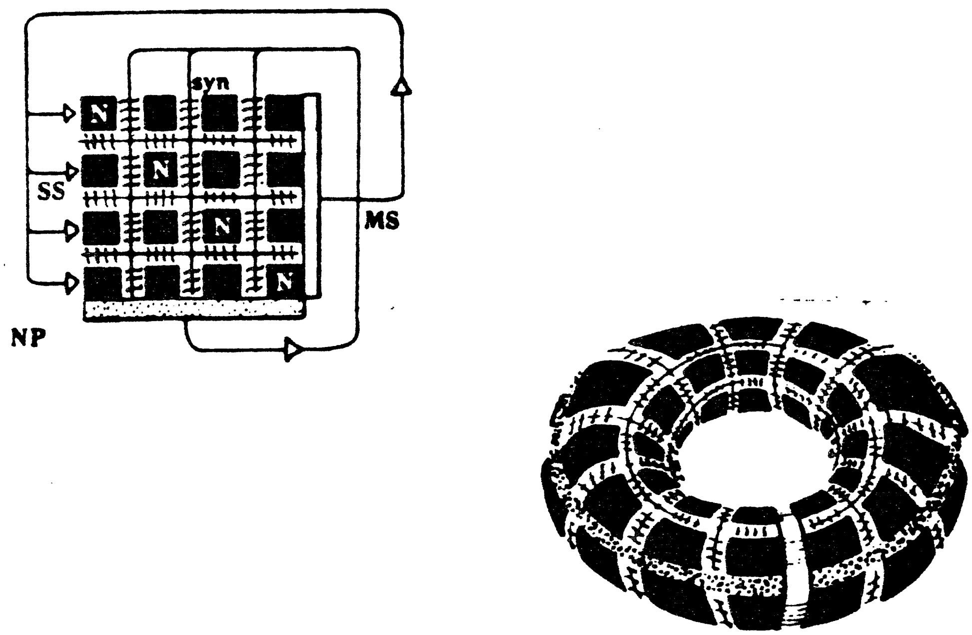


Figure 10

The black squares labelled N represent bundles of neurons that synapse with neurons of other bundles over the (synaptic) gaps indicated by the spaces between the squares. The sensory surface (SS) of the organism is to the left, its motor surface is to the right, and the neuropituitary(NP), the strongly innervated mastergland that regulates the entire endocrinal system, is the stippled lower boundary of the array of squares. Nerve impulses travelling horizontally (from left to right) ultimately act on the motor surface (MS) whose changes (movements) are immediately sensed by the sensory surface (SS), as suggested by the "external" pathway following the arrows. Impulses travelling vertically (from top to bottom) stimulate the neuropituitary (NP) whose activity releases steroids into the synaptic gaps, as suggested by the wiggly terminations of the lines following the arrow, and thus modify the *modus operandi* of all synaptic junctures, hence the *modus operandi* of the system as a whole. Note the double closure of the system which now recursively operates not only on what it "sees" but on its operators as well. In order to make this two-fold closure even more apparent, wrap the diagram around its two axes of circular symmetry until the artificial boundaries disappear and the torus is obtained. This is the functional organization of a living organism in a (dough)nut shell. <<This paragraph is taken from von Foerster (1973).>>

09. The closure theorem: "Eigenrelations arise in every operationally closed system."

Among the many variations and paraphrases of this amazing theorem, I have taken the Francisco Varela, Joseph Goguen version, because I see an affinity with the vocabulary of social science. Implicit in the word 'relation' are the concepts of 'behavior', 'conduct', and the recognition of regularity, of 'invariants', in the temporal course of an event. Here, among social scientists, one is not very interested in whether the cosine or sine appears as an eigenrelation, but rather whether, in the meeting of two persons, they greet each other with a handshake or with a bow.

One can go even further and look for the emergence of invariants, which arise if the air passes in certain ways through the vocal chords, whose oscillations produce whispers and grunts, by which two persons greet one another. In the south they say "Gruss Gott!" and in the north you will hear "Guten Tag!"

With everything that I have said until now, I have tried to show that these invariants, these "eigenrelations", are the essential form revealed from the recursive interchange of the participants of a social situation.

I would like to return to the original question: "How recursive is communication?" and to my suggestion:

00. Communication is Recursion.

With the vocabulary that we have developed I can sharpen this statement with a few words:

10. Communication is the eigenrelation in a recursively operating, double closure system.

The essential point of the topology of double closure is that it does not allow the pseudo solution of the hierarchy, by which one always pushed the responsibility to a higher level. Instead, through the bound heterarchical organisation, the operator becomes the operand and the operand the operator. This is exactly what we wanted to understand - that which in a one-dimensional logic would be made impossible. Through the interchangeability of the effects and the values of the pertinent functors, the freedom of dealing with a situation is given back to us and with it also our responsibility.

And with that I will close with thanks to Wilhelm Busch:

11. Wilhelm Busch's Desideratum:

"Two times two is four" is true
But it's too bad that it is easy and empty.
Because I would have preferred clarity
About that which is deep and difficult."

Whether I have succeeded, I don't know, but I thank you very much, that you have been so patient and kind to listen to me. And once more I would like to wish Niklas Luhmann well on his birthday.

References

Kauffman, Louis H. (1987), "Self-reference and recursive forms." *Journal of Social and Biological Structures*, Vol.10, pp. 53-72.

Luhmann, Niklas (1991), "Wie lassen sich latente Strukturen beobachten?", in Paul Watzlawick, Peter Krieg (Eds), *Das Auge des Betrachters: Beiträge zum Konstruktivismus. Festschrift für Heinz von Foerster*, Piper, München-Zürich, pp. 61-74.

McCulloch Warren S. (1945), "A Heterarchy of Values Determined by the Topology of Nervous Nets", *Bulletin of Mathematical Biophysics*, Vol.7, pp. 89-93; reprinted in Warren S. McCulloch, *Embodiments of Mind*, MIT Press, Cambridge, Mass. (1989), pp. 40-45.

Menger, Karl (1959), "Gulliver in the Land Without One, Two, Three", *Mathematical Gazette*, Vol. 43, pp. 241-250.

Menger, Karl (1962), "Function algebras and Propositional Calculus", in *Self-Organizing Systems*, Spartan Books, Washington, D.C., pp. 525-532.

von Foerster, Heinz (1973), "On Constructing a Reality", in *Environment Design Research*, Vol. 2, F.E. Preiser (Ed.), Sowden, Hutchinson and Ross, Stroudberg, pp. 35-46; reprinted in *Observing Systems*, Intersystems Publications (1981) , pp. 288-309.

Wittgenstein, Ludwig (1921), *Tractatus Logico-Philosophicus*, reprinted by Routlege and Kegan Paul, New York (1969).